

## Seminar: The Fargues–Fontaine curve

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In 2009 Fargues and Fontaine discovered the “fundamental curve of  $p$ -adic Hodge theory.” This is a 1-dimensional scheme over the  $p$ -adic numbers  $\mathbb{Q}_p$  which in many ways behaves like the projective line  $\mathbb{P}_{\mathbb{C}}^1$  over the complex numbers and thus deserves to be called a “curve.” It has by now become a central object of Arithmetic Geometry. For instance, it yields a geometric interpretation of  $p$ -adic Hodge theory: Closed points on the curve correspond to untilts of a fixed perfectoid field of characteristic  $p$ , Fontaine’s period rings appear naturally as rings of functions on the curve, filtered  $\phi$ -modules (a  $p$ -adic analog of mixed Hodge structures) give rise to vector bundles on the curve, etc.

The curve has already several applications in Arithmetic Geometry: The classification of vector bundles on the curve gives a short proof of Fontaine’s “weakly admissible  $\Rightarrow$  admissible”-conjecture (first proven by Colmez–Fontaine), which establishes an equivalence of categories between crystalline  $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ -representations and weakly admissible filtered  $\phi$ -modules – objects of (semi)linear algebra. The curve yields a geometric interpretation of local class field theory and it allows to describe  $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$  as a geometric fundamental group. Finally, it is the central tool for work of Fargues and Scholze on geometrizing the local Langlands correspondence.

The curve has several incarnations: As a scheme, as an adic space, and as a diamond. While the theory of diamonds gives a very nice conceptual definition of the curve, our goal in the seminar will be to understand the construction of the curve as a scheme, the classification of vector bundles on it, and some applications. We will mainly follow the book [FF] and lecture notes [A]. We won’t assume knowledge of  $p$ -adic Hodge theory.

Time and place: Tuesdays 13-15 (P-701 or online).

If you are interested in the seminar, please contact the organizers, preferably before the start of the semester on October 26.

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Literature:

- [A] J. Anschütz, *The Fargues-Fontaine curve*, lecture notes, 2020, [https://www.math.uni-bonn.de/people/ja/thecurve/vorlesung\\_the\\_curve.pdf](https://www.math.uni-bonn.de/people/ja/thecurve/vorlesung_the_curve.pdf).
- [FF] L. Fargues, J.-M. Fontaine, *Courbes et fibrés vectoriels en théorie de Hodge  $p$ -adique*, Astérisque 406, 2018, [https://webusers.imj-prg.fr/~laurent.fargues/Courbe\\_fichier\\_principal.pdf](https://webusers.imj-prg.fr/~laurent.fargues/Courbe_fichier_principal.pdf).