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Lecture + Seminar WiSe 20/21

## Operators on infinite graphs

Graph structures play an immense role in science and technology. For instance, in (mathematical) physics, graphs serve as a model with nodes representing possible positions of an electron and with edges (connections between the nodes) representing its possible paths of travel. These dynamic processes can in part be described by spectral properties of operators over discrete spaces. Graphs give rise to many beautiful mathematical phenomena - some of which become especially interesting when dealing with infinitely many nodes.

The goal of this course is to introduce such infinite graphs as an analytic object and to study the (in many cases unbounded) Laplace and Schrödinger operators arising from them. A special focus will be on the interplay between the geometric and analytic properties of the given graph and spectral theoretic features of the associated operator. For our studies we have a great variety of topics to choose from, such as the existence of solutions to the Schrödinger equation for energies below the bottom of the spectrum, aspects of the heat flow on the graph and recurrence phenomena on large graphs.

The class is designed as a combination of a lecture and a seminar. There will be two lectures per week until the Christmas break. Afterwards the class resumes in seminar mode, where the participants present talks on subject-related topics. There will also be the option to work together in pairs. The seminar topics will be handed out after four weeks of classes.

Students interested in the course should be familiar with or eager to learn about spectral theory of unbounded operators in Hilbert spaces. However, I will give a short introduction of these concepts during the first weeks and will provide supplementary reading material for students who wish to catch up or refresh their knowledge. For the sake of getting additional intuition from a finite-dimensional perspective I also recommend the lecture “Analysis und Geometrie endlicher Graphen” (in German) by Dr. Marcel Schmidt that you could take in parallel.

**Target group:** diploma and master students

**Prerequisites:** basic knowledge on functional analysis, in particular spectral theory of unbounded linear operators on Hilbert spaces (supplementary reading material will be provided)

**Mode:** The course can be taken as an online course. Within the framework given by the current regulations at the time we will arrange for

as much classroom teaching as possible, in particular during the seminar phase. If you are interested in taking the course please enroll in the moodle2 course (that will be set up soon). Further information will be given there in due time.

**Schedule:** starting Oct 27th on Tue 15:15-16:45 and Wed 11:15-12:45 in SG 3-12 (in classroom phases)

## Literatur

- [1] Matthias Keller, Daniel Lenz and Radoslaw Wojciechowsky. *Graphs and discrete Dirichlet spaces* book in preparation, 2021+.
- [2] Rainer Nagel. *A short course on operator semi-groups*. Springer Verlag Berlin Heidelberg New York, 1998.
- [3] Michael Reed and Barry Simon. *Methods of Modern Mathematical Physics I, II and IV*. Academic Press New York and London.
- [4] Konrad Schmüdgen. *Unbounded Self-adjoint Operators on Hilbert Space*, GTM Springer-Verlag.
- [5] Joachim Weidmann. *Lineare Operatoren in Hilberträumen I: Grundlagen* (for German speakers). B.G.Teubner Stuttgart Leipzig Wiesbaden (in German).
- [6] Dirk Werner. *Funktionalanalysis* (in German). Springer-Verlag.
- [7] Wolfgang Woess. *Random Walks on Infinite Graphs and Groups*. Cambridge University Press, New York, 2000.